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STEERING CONTROL WITH FEEDBACK DELAYS FOR BALANCING A MOTORCYCLE AT ZERO LONGITUDINAL SPEED

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ABSTRACT

The stability of a riderless motorcycle is investigated. The linearized governing equations are derived analytically with Kane's method. A simple linear state feedback controller with feedback delays is designed in order to stabilize the motorcycle at zero longitudinal speed using the steering mechanism. The linear stability properties are analyzed numerically, namely, we use semi-discretization to construct the stability charts of the delayed feedback controller. It is shown that the feedback delays significantly restrict the stable domain of the control gains.

Keywords: Motorcycle, Balancing, Control, Time Delay

1. INTRODUCTION

Balancing single-track vehicles such as bicycles or motorcycles is a complex task at zero longitudinal speed, which provides challenges for engineers. To begin with, a spatial mechanical model is required to describe the motion of the vehicle. In addition, the nonlinear governing equations have a complex structure that also involves geometric and kinematic constraints. Therefore, studies in the literature are often limited to linear semi-analytical or numerical analyses, i.e., the equations of motion are linearized around the equilibrium and only small vibrations are investigated.

In this study, we focus on the linear stability of a riderless self-driving motorcycle, considering zero longitudinal speed. Based on the idea of Honda Riding Assist [1], we examine the case when the vehicle is balanced by its steering mechanism.

For this, the mechanical model of the motorcycle is constructed based on the Whipple model [2] and the equations of motion are derived with the help of Kane's method. Thus, the kinematic constraints are eliminated by the introduction of the so-called pseudovelocities. Then, we design a steering controller to stabilize the motorcycle in the vertical position. A linear state feedback control law is applied, in which the lean and the steering angles/rates are taken into account. For these states, we consider different time delays in the control loop and we analyze the effect of these delays by means of linear stability charts.

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2. MECHANICAL MODEL AND GOVERNING EQUATIONS

According to the Whipple bicycle model [2], the motorcycle consists of four rigid bodies: the chassis, the fork, the front and the rear wheels, see Fig. 1(a). The wheels are modeled with rolling disks having single contact points with the ground. The steering geometry is characterized by the trail e, the rake angle ε and the fork offset d, see Fig. 1(b).

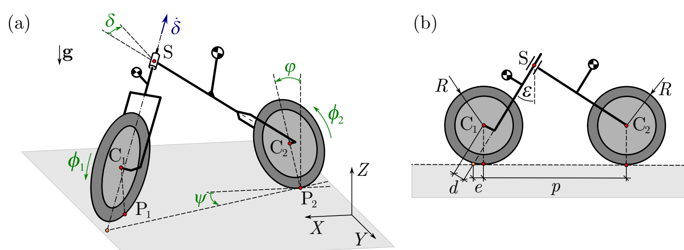

FIGURE 1: SPATIAL MECHANICAL MODEL AND THE RELEVANT GEOMETRIC PARAMETERS IN SIDE VIEW

The configurational space is seven-dimensional [3], i.e., one has to choose seven generalized coordinates for a unique description. Let us choose X and Y as the coordinates of the center point C_2 of the rear wheel, ψ as the yaw angle, φ as the lean angle, δ as the steering angle, ϕ_1 and ϕ_2 as the rotational angles of the front and the rear wheels around their own axes, see Fig. 1(a).

Considering the pure rolling of the wheels, four scalar kinematic constraining equations can be formulated, one longitudinal and one lateral for each wheel. The rotational speed of the rear wheel is also constrained: $\dot{\phi}_2 = v/R = 0$, since our goal is to stabilize the motorcycle for zero longitudinal speed ($v = 0$). Due to the presence of the kinematic constraints, the system is nonholonomic and the governing equations can be derived e.g. with

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Kane's method [4]. In order to have a unique description, we choose the lean rate and the steering rate as pseudovelocities (i.e., $\sigma_1 = \dot{\varphi}$ and $\sigma_2 = \dot{\delta}$).

As a result, for zero longitudinal speed, the linearized equations of motion can be written as

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{Q}\,, \tag{1}$$

where $\mathbf{M}$ is the mass matrix, $\mathbf{K}$ is the stiffness matrix, $\mathbf{x} = \begin{bmatrix}\varphi & \delta\end{bmatrix}^{\mathrm{T}}$ and $\mathbf{Q} = \begin{bmatrix}0 & M^{\mathrm{s}}\end{bmatrix}^{\mathrm{T}}$ with the internal steering torque M^{s}.

3. CONTROL DESIGN AND RESULTS

To stabilize the motorcycle using the steering mechanism, we use a simple linear state feedback controller:

$$M^{\mathrm{s}} = -P^{\mathrm{s}}_{\varphi}\varphi(t-\tau_{\varphi}) - P^{\mathrm{s}}_{\delta}\delta(t-\tau_{\delta}) - D^{\mathrm{s}}_{\varphi}\dot{\varphi}(t-\tau_{\varphi}) - D^{\mathrm{s}}_{\delta}\dot{\delta}(t-\tau_{\delta})\,, \tag{2}$$

where P^{s}_{φ}, P^{s}_{δ}, D^{s}_{φ} and D^{s}_{δ} are proportional and derivative control gains related to the lean and the steering angles, respectively. The time delays τ_{φ} and τ_{δ} are introduced to consider different sensor, computational and actuation delays for the lean and the steering angles/rates.

When the feedback delay related to the steering is neglected, i.e., $\tau_{\delta} \approx 0$, the control law in Eq.(2) is equivalent to the hierarchical control of our previous study [5]. There, it was shown that having a negative trail ($e < 0$) is beneficial for the balancing task. Therefore, we apply a negative trail in this present analysis, as well.

Let us consider the case when none of feedback delays is neglected, i.e., $\tau_{\delta} > 0$ and $\tau_{\varphi} > 0$, two of the control gains are fixed, i.e., $P^{\mathrm{s}}_{\delta} = 100\,\mathrm{Nm}$ and $D^{\mathrm{s}}_{\delta} = 10\,\mathrm{Nms}$, and the geometric parameters are based on a small-scale experimental rig [6]. The stability charts can be constructed with the help of the semi-discretization method [7] and stability boundaries can be plotted in the plane of control gains P^{s}_{φ} and D^{s}_{φ}.

The effect of τ_{δ} on the linear stability properties can be shown by calculating the stable domain of the control gains for different τ_{δ} values, while τ_{φ} is fixed. In the stability charts of Fig. 2, the stable domains are shaded. As it can be observed, the larger the steering feedback delay is, the smaller the stable domain of the control gains is. Moreover, as the delay is increased, a new horizontal stability boundary appears and bounds the stable domain. Even a small increase in the delay restricts the stable domain to a great extent. However, for $\tau_{\delta} < 0.1$ ms, the steering feedback delay has no effect on the size of the stable domain.

4. CONCLUSIONS

The balancing of a riderless motorcycle was investigated, considering the Whipple model with zero longitudinal speed. A linear state feedback controller was constructed with two feedback delays. It was shown that a critical and an ultimate value of the steering feedback delay τ_{δ} can be determined. A critical value exists below that the steering delay has no effect on the stable control gains domain. When the delay exceeds an ultimate value, no stable domain remains for the control gains.

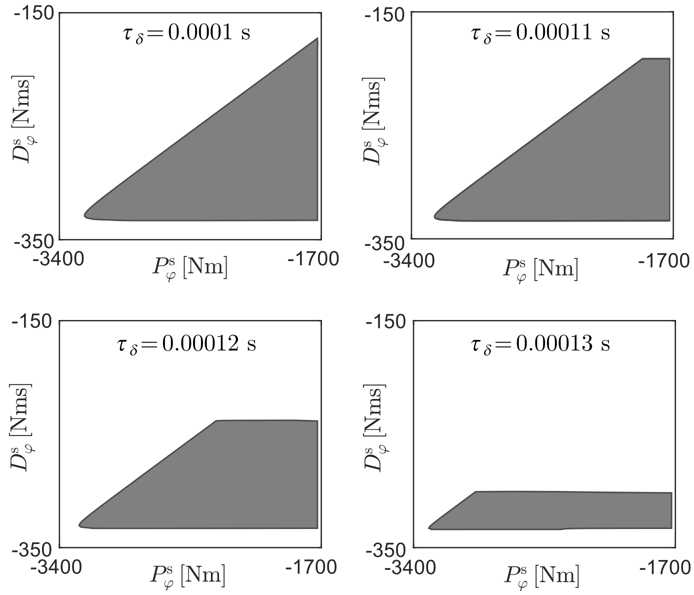

FIGURE 2: STABILITY CHARTS FOR FIXED LEAN FEEDBACK DELAY τ_{φ} = 0.0001 s AND FOR DIFFERENT VALUES OF STEERING FEEDBACK DELAY τ_{δ}

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